Theoretical Understanding of Transport in Fusion Plasmas

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Abstract

In this paper the theoretical understanding of transport in fusion plasmas is discussed. A few examples of key effects that are now understood are being presented. The theory is discussed in words without many details.

1. Introduction

Fusion has reached a crucial point in its development. A design has been made in which for the first time the energy production through fusion reactions should largely exceed the external heating, proving the feasibility of a fusion reactor. Although theory has played some role in reaching this design, many predictions are based on empirical scalings. Nevertheless, theory is making good progress in explaining some of the basic observations in fusion relevant plasmas, and is expected to play a more prominent role in the design of future machines. This paper discusses our understanding of transport in fusion plasmas on the basis of some well chosen examples.

The main practical goal in fusion research is to provide an energy source for the future. This should be accomplished through the controlled fusion of light elements releasing large amounts of binding energy. Because of its most favourable cross section at moderate (10 keV) temperatures the reaction between Deuterium and Tritium is currently favoured

$$D + T = 4He + n + 17.6 \text{ MeV} .$$

Although Tritium has a relatively short life time and does not occur in large quantities in nature it can be bread using the neutron produced in this reaction to cause a second reaction with a Lithium atom. There is, therefore, a large amount of fuel available in nature. Due to the large coulomb interaction it is, however, difficult to obtain this reaction in a controlled way. Temperatures of the order of 10 keV are needed (at which the matter is fully ionized, i.e. a plasma), which due to their large thermal velocities can rapidly escape from the plasma. Two concepts are being investigated, inertial confinement and magnetic confinement. In this paper only the latter will be discussed.

The idea behind magnetic confinement is simple enough. If a magnetic field is applied then the motion of the charged particles perpendicular to the field is limited to the radius of the Larmor orbit. This for typical reactor parameters is only a few mm, whereas the device has a size of several meters. Therefore the particles are well confined perpendicular to the magnetic field. Still, of course, the motion along the magnetic field is not hindered by the Lorentz force, and one must use a field configuration in which the field lines go around but never touch the wall.

Because of the divergence free nature of the magnetic field, the simplest configuration in which this can be achieved has the shape of a donut with a symmetry around the main axis. This concept is used in the tokamak, which is the most developed device to obtaine controlled fusion. The tokamak is shown in Fig. 1. Its magnetic field is generated from the outside using toroidal field coils as well as internally through the use of a plasma current which generates a poloidal magnetic field. The field lines therefore wind helically around the torus. Best confinement is achieved if these magnetic field lines map out nested toroidal surfaces, the so called flux surfaces.

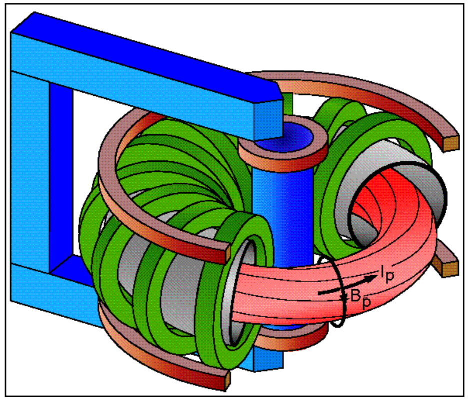


Fig. 1: The tokamak concept

Besides the practical goal, fusion research has a large spin off in high temperature plasma physics. The parameters needed for fusion (density 10^{20} particles per m^3 and temperatures of around 10 keV embedded in a magnetic field of several Tesla) are unique in man made devices. They do occur, however, frequently in astro-physical systems, which of course are much more difficult to observe, not too speak of influence. Theory developed and tested for fusion plasmas is of great importance to astro-physics.

There has been a continuous effort in fusion research for many decades. The first results were disappointing because the radial transport of energy and particles was much larger than expected. Nevertheless, in the Joint European Torus (JET) one has now reached the point of break-even, i.e. the plasma parameters are such that the same amount of energy would be produced through fusion reactions as is used through external heating. Part of the research on fusion is done in the Max-Planck-Institute for Plasma Physics in Garching near Munich. The tokamak available in this institute is ASDEX Upgrade, and several results of this machine will be shown.

Using the energy confinement results from the machines all over the world, it has been possible to make an empirical scaling. With this scaling the next device ITER has been designed. ITER (International Thermonuclear Experimental Reactor) is projected to produce ten times more power through fusion reactions than is used for external heating of the plasma. The physics of this experimental reactor design is well worked out, and it is now a political decision whether it will be build or not.

The fact that one has used empirical scaling laws to determine the energy confinement in ITER, already shows that this can not be calculated from first principles. The energy confinement is a long outstanding problem in fusion. For many decades it is believed to be caused through turbulent fluctuations that are generated by micro-instabilities. However, even a qualitative description of the observed phenomena was missing for many decades. This has changed in recent years, and some examples of this will be given in this paper.

There are far more problems that theory has dealt with and that are not discussed in the paper. Among them are large scale instabilities, heating and current drive, the effect of fast particles (for instance the alpha particles which have an energy of several MeV) on instabilities, scrape-off layer physics and plasma wall interaction. The research has a multi-diciplinary nature and it is difficult to give a complete overview in a paper like this. We will therefore concentrate on the effects described in the previous paragraph.

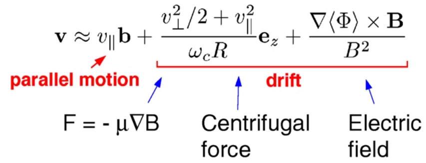
The remainder of the paper is structured as follows. The second section introduces the particle orbit theory necessary to understand the following sections. The next section then gives an example of the progress in understanding of the turbulent state and the energy confinement. Finally the possibility to use physics insights to improve the performance are discussed, after which a short conclusion is given.

2. Particle orbits

Because of the strong magnetic field used in fusion research the dominant force is the Lorentz force. This means that all forces perpendicular to the magnetic field are compensated by this force, which then leads to a (small) velocity of the plasma perpendicular to the field. Along the magnetic field the motion is free and because of the high temperature very fast. To understand the results of this paper one has to consider essentially three forces: the force due

to the electric field, the centrifugal force that appears through the combination of the fast parallel motion and the curvature of the magnetic field, and the mirror force (F = – magnetic moment times grad B) which is due to the change in the magnetic field. These three forces lead to a drift which for a tokamak can be written to be approximately

$$\mathbf{v} \approx v_\parallel \mathbf{b} + \frac{v_\perp^2/2 + v_\parallel^2}{\omega_c R}\mathbf{e}_z + \frac{\nabla\langle\Phi\rangle \times \mathbf{B}}{B^2}$$



The first term on the right hand side is the fast parallel motion. The second is due to the combination of centrifugal and mirror force and points approximately in the vertical direction along the axis of symmetry. The last term is due to the electric field and is perpendicular to the gradient in the electric potential.

The mirror force also works in the direction of the magnetic field. Due to the toroidal curvature the magnetic field in the tokamak increases towards the magnetic axis, and the mirror force, therefore, leads to trapping of some of the particles (those with low parallel velocity) in the magnetic well on the outboard side of the surface. These particles are particularly sensitive to the drift. One can, however, show that the vertical drift does not lead to the escaping of particles from the plasma. Although the particle drifts away from the magnetic surface while it is above the equatorial plane, it drifts back when it is below this plain. More generally one can show that this exact cancellation is directly connected with the toroidal symmetry. The canonical toroidal angular momentum is conserved from which it can be derived that the particles can only have a small excursion from the flux surface.

3. Transport and micro-instabilities

To understand the turbulent transport we have to investigate the nature of the micro-instabilities. These instabilities are driven by the thermodynamic forces, and the toroidal variants are made possible through the drift velocity. As an example the Ion Temperature Gradient (ITG) will be discussed here. This mode can be explained in the following way. Assume that an ion temperature perturbation exists on the outboard side of the flux surface, as shown in Fig. 2. The drift velocity due to the magnetic field inhomogeneity is in the vertical direction and it can be seen from the equation above that its magnitude depends on the energy of the particles. Therefore, it is on average larger in the hot regions compared to the cold regions. This means that the averaged drift velocity is modulated in the vertical direction and leads to a compression of the ions in the surface region. An ion density perturbation is generated that is 90 degrees out of phase with the temperature perturbation. Such a perturbation would suggest a charge separation, but this is in a plasma only possible on the small length scale of the Debye length. Basically, already small charge separations generate an electric field force that is very large and can not be balanced by any other force in the plasma. The plasma is therefore quasi-neutral and the electron density must follow the ion density. Here we will assume that the electrons react adiabatically, meaning that their response can be written as a Boltzmann factor containing the electric potential.

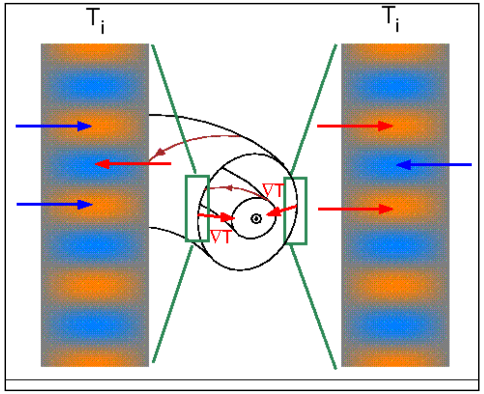


Fig. 2: Schematic picture of the ITG instability

The result of this approximation is that a potential is generated that is in phase with the density perturbation. This potential then leads in turn to an ExB drift which can be shown to be such that hot plasma is moved from the inside into the hot regions and cold plasma is moved into the cold regions. The initial perturbation is enhanced and the plasma is unstable. On the inboard side of the tokamak the temperature gradient points in the opposite direction and the mode is stable. Therefore, the toroidal variant of the ITG balloons on the outboard side of the flux surface. In general the size of the structures for the most unstable modes is of the order of the Larmor radius, which is much smaller than the machine size.

The modes that go linearly unstable grow until they are sufficiently strong to interact. At this point the plasma goes over in a turbulent state with many modes (both linearly unstable as well as stable ones) interacting. There are many turbulent systems in which the non-linear state has little to do with the linear state. Plasma turbulence in the core of the tokamak is however a state of weak turbulence. This means that the mode structures that appear in the non-linear state are largely the same as the linear state, and some of the properties of transport can be obtained from investigation of these mode structures, especially the phase relations between the different quantities. Of course to calculate the real flux one still has to estimate the saturation level of the instabilities. This is done in two ways, either by fitting to non-linear

simulations or through the so called quasi-linear theory. In the latter theory one assumes that the most unstable modes interact mostly with itself to generate the stabilization. Although this is a clear simplification, the fits to non-linear simulations are not available for models that contain all the necessary physics to describe particular situations.

Two other problems are of major importance. First, although the mode structures are largely unchanged, the dominant modes in the non-linear simulations do not necessarily have the same wave vector as the most unstable linear mode. In general one observes a shift towards larger wavelengths (although still within the range of the linearly unstable modes). This shift is not so large for the ITG, but can be larger in other cases. The second major problem is that in the non-linear stage large scale flows are generated which control (to some extend) the level of fluctuations, and therefore the transport fluxes. Such flows are never included in the linear models. From the discussion above it follows that the linear theory can never give us a complete picture of transport phenomena in tokamak plasmas. There is a strong need for non-linear computations. However, when applied carefully (with the guidance of non-linear theory) many physical phenomena can be understood on the basis of linear theory. These theories are often more transparent, and have furthermore the advantage that they can be applied to many more cases compared with the non-linear runs which are quite computer power demanding. A successful approach to the understanding of transport in tokamak plasma therefore incorporates both linear as well as non-linear investigations.

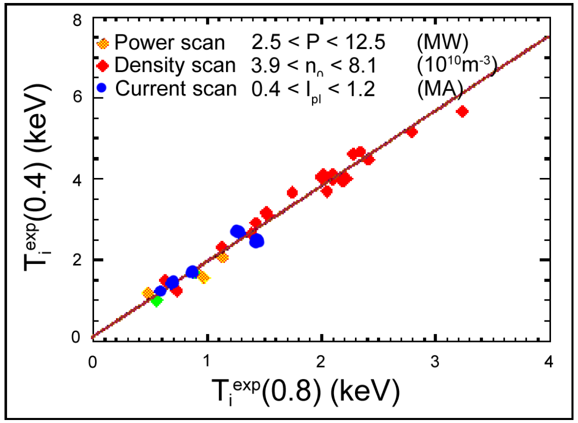


Fig. 3: Experimental central ion temperature against the temperature at the edge of the plasma for different scans in plasma parameters (density, current, power)

The ITG is dominantly driven by the ion temperature. And in the form in which it is explained above, it transports only ion heat. The description above, however, is not complete. If trapping of the electrons is introduced this mode transports also particles as well as electron heat. Trapping of the electrons makes a difference because the trapped particles can not move along the field line over the entire surface. This makes that they do not have the adiabatic response that was previously used. The description above is also incomplete in another way. One might have the impression that the mode is unstable regardless the value of the ion temperature gradient (as long as it is pointing inwards). Several effects, however, lead to a threshold for the mode. This threshold is a threshold in the logarithmic derivative of the ion temperature

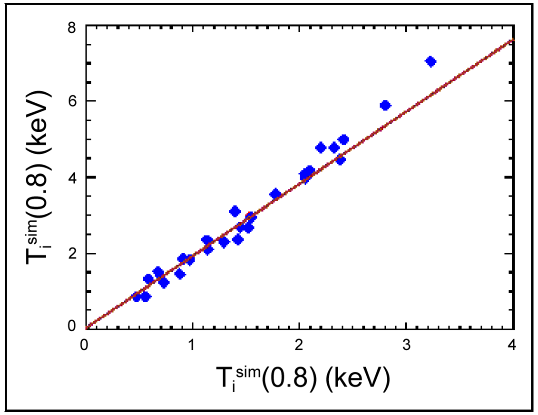


Fig. 4: Prediction of the ITG based models of the quantities also shown in Fig. 3. The line represents the experimental values

The toroidal ITG is a relatively strong instability. Above the threshold for the mode the transport increases strongly with R/L_t. Under experimental conditions, where the heating power is limited, this means that the threshold can not be exceeded by a large amount. If it would, then a large heat flux would be generated that relaxes the ion temperature profile towards the critical gradient on a short timescale. This leads to a particular prediction. Since it is the logarithmic derivative that is more or less fixed an integration over the radius shows that the temperature in the core of the plasma is proportional to the temperature of the edge (of course up to the point where this model can be applied which is roughly 80% of the radius). Increasing the edge temperature, which is much smaller than the core temperature, by a factor two will therefore also increase the central temperature by a factor two. This is of course in sharp contrast with a model with constant heat conduction coefficient in which the change in the core temperature would be simply that of the edge temperature (which is much smaller than the core temperature). This insight explains a long observed phenomena, namely that the temperature profiles are resilient. The profiles have a tendency to have the same shape independent of the position of the heating or the value of other plasma parameters. Graphically this can be shown by plotting the central ion temperature against the edge temperature, as shown in Fig 3. The model above would predict a straight line through the origin, which is indeed observed. The different quasi-linear models that are based on ITG

physics explain this behaviour also qualitatively relatively well. They allow for a prediction of the central ion temperature to within 20% when the boundary temperature is given (see Fig.4).

Besides the ion temperature profile also the electron temperature and particle density can be predicted relatively well, although not as good as the ion temperature. Electron heat is transported both by the ITG (if the influence of trapped electrons is considered) as well as the Trapped Electron Mode (TEM). Which of the two modes is dominant depends on the profiles of both temperatures as well as the density, and is to some extend controlled by which component is dominantly heated. Under dominant ion heating (and not too peaked density profiles) the ITG dominates and the electron heat transport is more or less proportional to the ion heat transport, i.e. not very sensitive to the electron temperature gradient. In this case the electron temperature gradient can be varied over a relatively large range. For this case the quasi-linear fluid models give a reasonable description of the experiment, although the deviation is larger than for the ion temperature. In the case of dominant electron heating (and not too high density) usually the Trapped Electron Mode dominates. This mode then has a threshold that depends on both the electron temperature gradient and the density gradient. Comparison of the linear growth rate of the TEM with the measured heat flux (using one arbitrary scaling parameter) agrees well. These comparisons point at a threshold as well as at a moderate stiffness for the electron channel (i.e. transport does not increase as strong above the threshold as for the ITG in these experiments, which is largely related to the often lower temperature). Both the ITG and TEM also transport particles which form a special case since they often exhibit an inward pinch, leading to peaked density profiles even in the absence of a central particle source. To describe this pinch well, another ingredient has to be included: the collisions. Again some features of the density profile can be explained qualitatively.

The above is a very short (and incomplete) summary of what is understood.

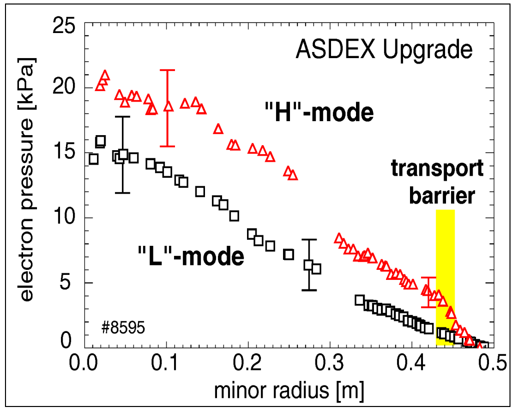


Fig. 5: Electron pressure profiles for the L- and the H-mode

It is important to point out that the models discussed here do not yet have a predictive power. This is due to the fact that they all use the measured edge temperatures and densities as boundary condition. The edge is a special case, because different instabilities dominate. In this region the turbulence is strong and the linear analysis lead to the wrong conclusions. The most famous example of this is the non-linear drift wave instability. In this cases the modes are linearly stable, but non-linearly unstable. It is clear that in this case any linear analysis fails completely. There is however, a second reason why the edge is difficult to describe. There are two basic modes of operation in the tokamak, the Low confinement mode (L-mode) and the High-confinement mode (H-mode). The latter distinguishes itself from the first through the existence of a steep gradient region in the edge of the plasma (for both temperatures and density) referred to as 'transport barrier'. The two different modes are shown in Fig. 5. Although the reason for the occurrence of this barrier is generally accepted to be large shear flows, there is no good understanding of how these are generated, and consequently there is no good prediction of the boundary conditions on the top of the steep gradient region.

4. Advanced concepts

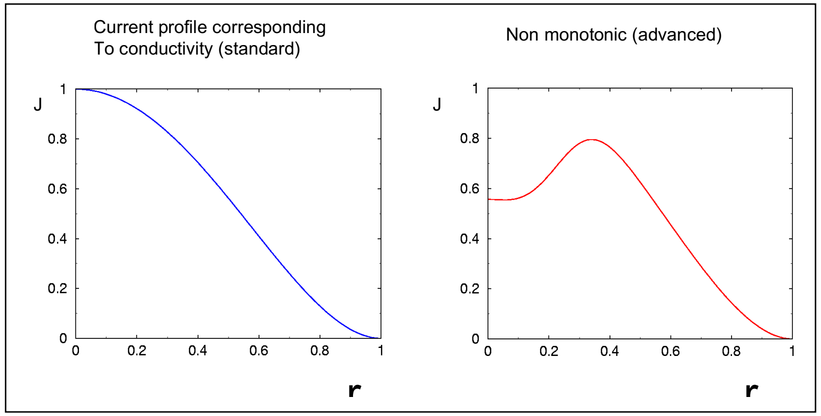


Fig. 6: 'Standard' and 'advanced' current density profiles

Advanced scenarios in the tokamak aim at an improvement of confinement and stability over the current predictions made for a reactor. Several improved scenarios are being discussed, but we will concentrate here on the internal transport barriers. In this scenario one aims at generating a region of improved confinement inside the plasma much in the same way as the H-mode barrier at the edge. Not surprisingly, also in this scenario large shear flows are expected to be important. Other than the H-mode the internal barriers, however, do not form spontaneously. Special conditions are necessary for it to occur. The shear flows under normal

conditions are not sufficiently strong, and other stabilizing effects on the mode are necessary to reduce the growth rate of the instability such that the shear flows can suppress them. This can be done through changes in the magnetic topology. Because of the current in the plasma the field lines are helical, and this helicity is measured with the so called safety factor, q. This safety factor is essentially the number of turns a field line makes in the toroidal direction during one complete poloidal turn. The safety factor varies as a function of radius due to the radial profile of the plasma current. And therefore the pitch of the magnetic field, which is measured by the magnetic shear, s, changes. Under normal circumstances the shear in a tokamak is positive because of the peaked current profile. If one however makes a hollow current profile the shear becomes negative over some region of the plasma, as shown in the Fig. 6. This has a profound influence on the instabilities.

The reason why the shear matters for the micro-instabilities, is that the dynamics parallel to the magnetic field can not be neglected. In our explanation of the ITG we took a 2D case localized on the low field side. The frequencies that we find for these modes, however, are comparable to the transit time of the ions along the magnetic field. This means that the structures in temperature and potential in fact are not local but extend along the magnetic field, and therefore feel the pitch of the field lines. Figure 7 shows how these structures change starting with a elongated structure at the low field side and propagating a certain distance along the magnetic field. Of course they are displaced in the poloidal direction due to the helical nature of the field lines. But since they have a radial extend and the pitch of the field line is different the displacement is not the same on every radial surface. It turns out that for positive shear the tilting of the structure is such that the structure remains more horizontal, whereas the tilting for negative shear is such that the structure becomes more vertical. Since the drift velocity is in the vertical direction, and since the compression of the particles due to the vertical drift is essential in the drive of the mode, the positive shear is more unstable compared with the negative shear.

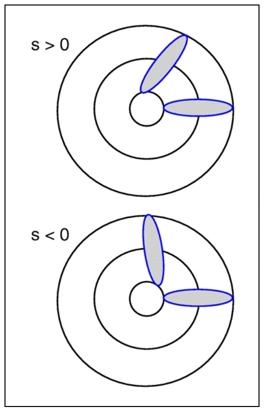


Fig. 7: Influence of the magnetic shear

Negative shear, however, does not occur naturally in the plasma. Currently it can be generated in the current ramp up phase. The current needs some time to diffuse into the plasma leading to hollow current profiles during and shortly after the current ramp up. Heating in the current ramp up phase can therefore be used to generate an internal transport barrier. The effect of this is shown in Fig 8. It can be seen comparing the temperature profiles of the L-mode and the internal transport barrier that the latter are much more steep in a localized region in the plasma, and consequently have a much better confinement. The magnetic shear as well as the velocity shear are thought to play an important role in the occurrence of this barrier but are surely not the only stabilizing mechanisms; the shift of the surfaces, the temperature ratio, as well as non-thermal ion populations are stabilizing as well. Also one should not have the impression that everything is perfectly understood. The triggering of the barrier is often not quantitatively explained by theory. Barriers propagate outward through a mechanism that is not perfectly understood.

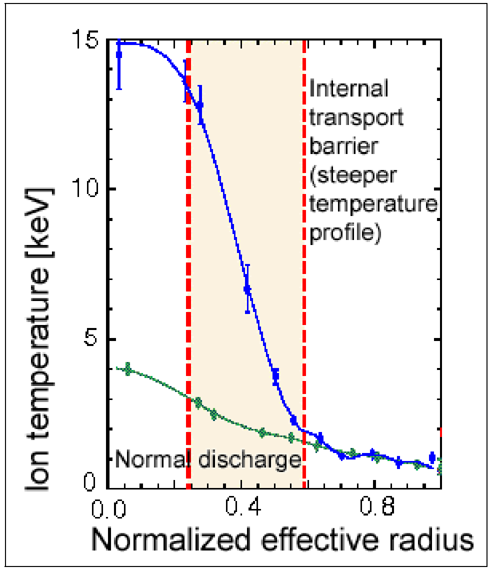


Fig. 8: Temperature profile of a transport barrier

Still the barriers could provide an attractive alternative for the operation of the reactor. The reason is that the improved confinement allows for the operation at lower current where a large fraction of the current could be provided by the so called bootstrap current. This current is a current that is generated by the radial gradient of the plasma pressure and must therefore not be provided through external means. It has the additional advantage that it is peaked off-axis and can therefore naturally provide for the hollow current profiles. It is hard to test such scenarios in current devices, and it must be said that the current experiments are far away

from convincingly showing the possibility of such a scenario. Nevertheless, this research is worth continuing since it could provide a steady-state tokamak reactor.

So far we have discussed only the results from tokamaks. This device is indeed the most advanced in terms of parameters that have been reached. But alternative devices have been suggested and built. The most prominent alternative is the stellarator, in which both the toroidal as well as the poloidal magnetic field are imposed from the outside. This is of course only possible in a non toroidally symmetric device. The advantage of a stellarator is that it can be operated in steady- state since no current has to be induced with a transformator coil like is the case in a tokamak. Furthermore, the current is a source of free energy which can lead to large scale instabilities, which are absent in the stellarator.

One of the disadvantages that the stellarator has had for many years though is that it did not confine the plasma well. Since there is no toroidal symmetry the confinement of the tokamak, in which the particles drift away from and back to the magnetic surfaces, no longer holds. For arbitrary fields the trapped particles would drift out of the machine and a reactor would not be possible. In this area theory has played an important role. It has designed magnetic field configurations that can be imposed form the outside with field coils and that do confine the particles. Essentially there are two ways to accomplish this. One is through the use of a quasi-symmetry. It can be shown that the particle orbits in the proper coordinate system depend only on the magnetic field strength and its gradients. Furthermore, if in this coordinate system the magnetic field has a symmetry the particle orbits are confined. One can choose either toroidal symmetry (tokamak like) or a helical symmetry.

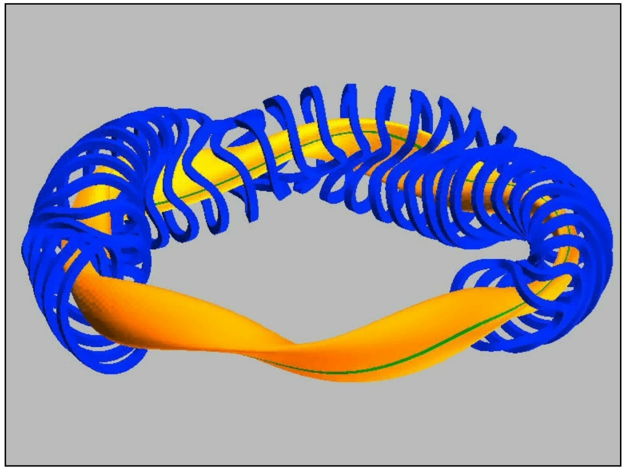


Fig. 9: W7X design

The second possibility to confine the particles sufficiently well is the use of the insight that can be obtained from the drift velocities. The most dangerous particles are the trapped particles because they do not move along the field and therefore do not average the drift velocity. If one makes the magnetic field configuration such that the particles are trapped in regions in which the drift velocity is small (or lies in the flux surface) good confinement can be obtained. This concept is used in the design of the W7-X stellarator which is currently being build in Greifswald, and is shown in Fig. 9. The simple explanation of how this machine works is as follows. The device essentially exists of 5 straight pieces that are put together. The radius of curvature of the magnetic field, and consequently the drift velocity, is large only in those regions in which one goes from one straight piece into the other. The magnetic field is furthermore chosen such that it is strong when the curvature is large. This means that the trapped particles are confined in the straight pieces where they undergo a small drift (that essentially lies in the flux surface). This design that relies heavily on a theoretical description should allow the stellarators to reach the same parameters as the tokamak, and should prove that they could indeed be a good alternative.

5. Conclusions

This paper gives only a few, and not very detailed examples, of our progress in the understanding of transport in fusion plasmas. Since it has been the challenging problem for decades the results are encouraging. It is expected that the combination of large computer simulations and insights into the physics of the modes, will reveal more effects in the near future.